Physics 201 48 pts Name Key

Exam 1 – Electrostatics

February 16, 2010

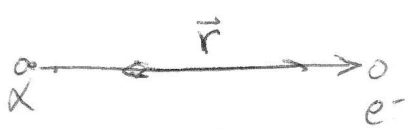
This is a closed book examination but during the exam you may refer to a 3"x5" note card with words of wisdom you have written on it. There is extra scratch paper available. Please explain your answers. Your explanation is worth 3/4 of the points on all questions.

A general reminder about problem solving:

- Focus
  - Draw a picture of the problem
  - What is the question? What do you want to know?
  - List known and unknown quantities
  - List assumptions
- Physics
  - Determine approach – What physics principles will you use?
  - Pick a coordinate system
  - Simplify picture to a schematic (if needed)
- Plan
  - Divide problem into sub-problems
  - Modify schematic and coordinate system (if needed)
  - Write general equations
- Execute
  - Write equations with variables
  - Do you have sufficient equations to determine your unknowns?
  - Simplify and solve
- Evaluate
  - Check units
  - Why is answer reasonable?
  - Check limiting cases!
- Show All Your Work!

The next two questions concern an electron (charge $-q_e$) and an alpha particle (charge $+2q_e$) that are separated by 16 nm in a region of space without any other charges.

1. [4 PTS] Compare the electrostatic force on the alpha particle, F_α, and the force on the electron, F_e.
   a) $4F_e=F_\alpha$
   b) $2F_e=F_\alpha$
   c) $F_e=2F_\alpha$
   d) $F_e=4F_\alpha$
   e) none of these



$$\vec{F}_\alpha = k\frac{q_\alpha q_e}{r^2}\hat{r} \qquad \vec{F}_e = -k\frac{q_e q_\alpha}{r^2}\hat{r}$$

Explain:

$|F_e| = |F_\alpha|$

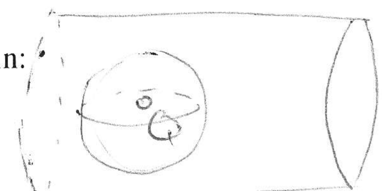
2. [4 PTS] The electron and alpha particle are moved apart so they are now separated by 33 nm.
   a) F_e increases and F_α decreases
   b) F_e decreases and F_α increases
   c) Both F_e and F_α decrease
   d) Both F_e and F_α increase
   e) none of these

$F \propto \frac{1}{r^2}$ so force decreases

Explain: $|F_e| = |F_\alpha|$ in fact $\vec{F}_e = -\vec{F}_\alpha$

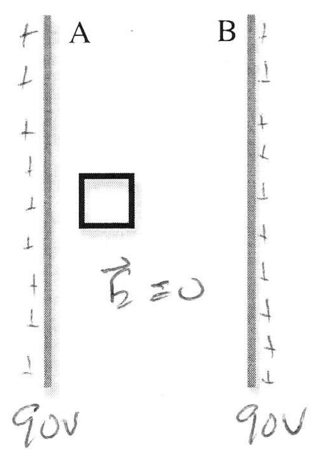
3. [4 PTS] A single point charge (Q) is located at the center of an imaginary sphere of radius 1m and a much larger imaginary cylinder of diameter 2 m and side length 10 m. Compare the electric flux through each.
   a) The electric flux is zero through both the sphere and cylinder.
   b) The magnitude of the electric flux is greater through the sphere.
   c) The magnitude of the electric flux is greater through the cylinder.
   d) There is the same positive electric flux through both the sphere and cylinder.
   e) There is the same negative electric flux through both the sphere and cylinder.
   f) None of these.

   Explain:



Both volumes inclose Q so since $\Phi_E = \frac{Q_{enc}}{\epsilon_0}$ flux is the same and positive

The next two questions concern a hollow metal cube that is placed between two large charged plates. Both plate A and plate B are held at 90 volts. The plates are separated by 90 cm and the metal cube is placed 30 cm from plate A (so the cube is closer to plate A).



4. [4 PTS] The potential on the surface of the metal cube
   a) is 120 volts.
   b) is 90 volts.
   c) is 30 volts.
   d) must be zero.
   e) can not be determined. More information is needed.

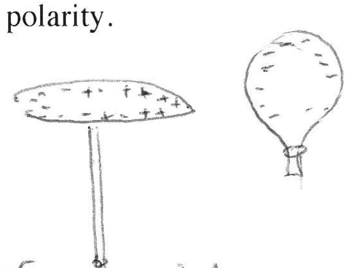
   Explain: The potential is constant between the two plates

5. [4 PTS] The electric field inside the metal cube
   a) is proportional to $1/r^2$
   b) is positive pointing towards plate A.
   c) is zero.
   d) is positive pointing towards plate B.
   e) can not be determined without the size of the cube.

   Explain: $\vec{E} = -\frac{dV}{dr}$ Voltage is constant so $\vec{E} = 0$

6. [4 PTS] A large neutral metal disk is placed on an insulating post. A negatively charged balloon is brought near it - but does not touch it. The balloon is taken away. The disk is now
   a) charged but we cannot know its polarity.
   b) neutral (has no net charge).
   c) negatively charged.
   d) positively charged.
   e) none of these.

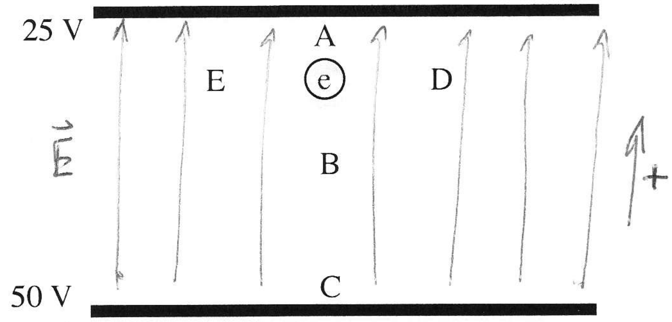
   Explain:



Balloon Near ⇒ induces charge to move but $\sum q_{disk} = 0$ (neutral)

Balloon far - charges redistribute

7. [4 PTS] An electron is placed between two large plates held at different voltages. The electron is placed very close to the plate that is held at 25 volts.
   a) The electron moves towards A.
   b) The electron moves to the middle (position B).
   c) The electron moves toward C.
   d) The electron oscillates between E and D.



Explain:
Electric field is uniform between the two plates so $F = qE$ is negative ∴ Electron moves in opposite direction to $\vec{E}$ towards "C" and the 50 V plate

The next two problems can be done on the back of your exam or on additional paper.

8. [10 PTS] An electron that is initially at rest is placed between two parallel plates. At time t = 0 sec an electric field (0.5 kN/C) is turned on between the plates. Note: q_e=-1.6x10^{-19} C and m_e=9.1x10^{-31} kg.
   a) What is the velocity of the electron after it has traveled 30 cm?
   b) How long does it take to travel 30 cm? NOTE: Think kinematics.

9. [10 PTS] A ball of negative charge has a constant charge density, 7.6x10^{-3} C/m^3. The ball has a radius R_B = 30 cm.
   a) Draw a graph of the electric field inside and outside the sphere?
   b) What is the potential difference between r_1 = 10 cm and r_2 = 20 cm?
   c) What is the potential difference between r_1 = 40 cm and r_2 = 60 cm?

Possibly useful mathematical relationships:

Law of Cosines $c^2 = a^2 + b^2 - 2ab\cos(\theta)$ which for θ=90° is the Pythagorean theorem $c^2 = a^2 + b^2$

Trigonometric identities:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$
$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$
$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$$

Derivative $\frac{d}{du}Cu^n = nCu^{n-1}$ and anti-derivative (integral) $\int Cu^n du = \frac{1}{n+1}Cu^{n+1} + const.$ of a polynomial

Derivative $\frac{d}{du}k\sin(au) = ka\cos(au)$ and integral $\int k\sin(au)du = -\frac{k}{a}\cos(au) + const.$ of the sine function

Derivative $\frac{d}{du}k\cos(au) = -ka\sin(au)$ and integral $\int k\cos(au)du = \frac{k}{a}\sin(au) + const.$ of the cosine function

The Chain Rule $\frac{d}{dz}f(u) = \frac{d}{dz}u\frac{d}{du}f(u)$

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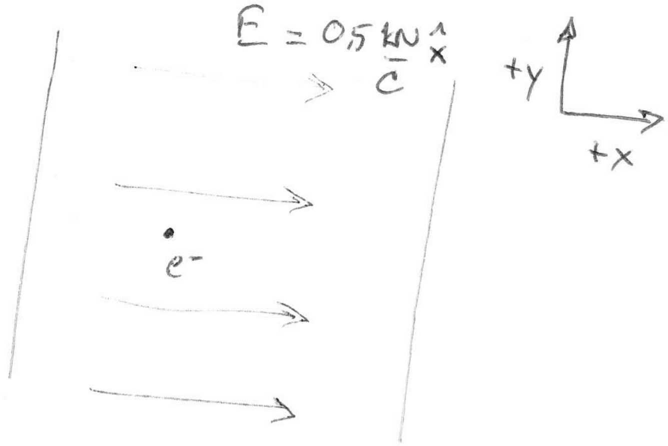


$\vec{F}_x = q\vec{E}$ Electron will move in $-\hat{x}$ direction

$F_y = 0$

$a_x = \frac{F_x}{m}$ $\quad v_x(t) = \int_{t_0}^{t_2} a_x \, dt \quad x(t) = \int_{t_0}^{t_2} v(t) \, dt$

$v_x(t) = v_0 + at \quad x(t) = x_0 + v_0 t + \frac{1}{2}a_x t^2$

substitute for time in position equation and solve for Δx

$x(t) - x_0 = \Delta x = \frac{[v(t)]^2 - [v_0]^2}{2a_x}$ or $v(t)^2 = 2a_x\Delta x + v_0^2$

(a) $v_0 = 0 \, m/s \quad \Delta x = 0.3 \, m$

$(v(t))^2 = 2a_x \Delta x \qquad a_x = \frac{F_x}{m} = \frac{qE}{m}$

$v = \left(\frac{2qE\Delta x}{m}\right)^{1/2}$

$\boxed{v = 7.3\times10^{6} \, m/s}$ 2.4% speed of light $c = 2.99\times10^{8} \, m/s$

(b) $v = v_0 + at \quad v_0 = 0 \, m/s$

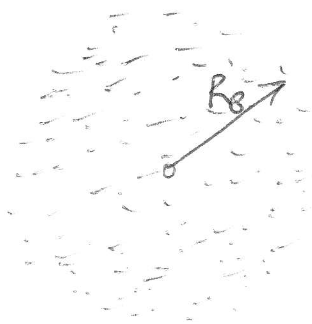
$t = \frac{v}{a} = \left(\frac{2qE\Delta x}{m}\right)^{1/2} \cdot \frac{m}{qE} = \left(\frac{2m\Delta x}{qE}\right)^{1/2}$

$t = \cancel{8.8\times10^{-10}} \, s = \cancel{0.88} \, ns$

$8.3\times10^{-8} \, s \quad \boxed{83 \, ns}$

$\boxed{9}$

$R_B = 30\ \text{cm}$



$\rho(r) = \rho_0 = 7.6\times10^{-3}\ \dfrac{C}{m^3}$

Use Gauss' ~~Law~~ to find $\vec{E}$

$$\oint \vec{E}\cdot d\vec{A} \equiv \Phi_E = \frac{Q_{enc}}{\epsilon_0}$$

choose spherical symmetry

Inside

$r \le R_B$

$$E\cdot 4\pi r^2 = \frac{1}{\epsilon_0}\int_0^r \rho_0 4\pi r^2\,dr = \frac{4\pi\rho_0}{\epsilon_0}\int_0^r r^2\,dr$$

$$E\,\cancel{4\pi}r^{\cancel{2}} = \frac{\cancel{4\pi}\rho_0}{\epsilon_0}\frac{r^{\cancel{3}}}{3}$$

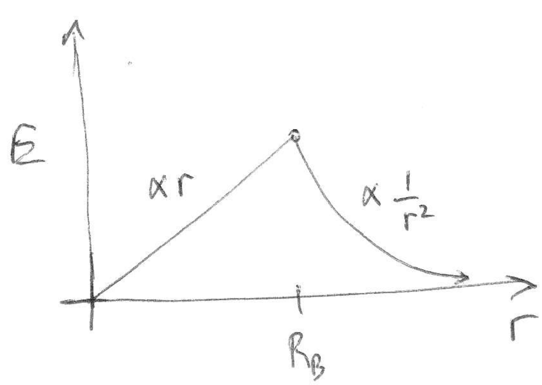
$$\vec{E} = \frac{\rho_0}{3\epsilon_0} r\,\hat{r} \qquad V_{in} = -\int \vec{E}\cdot d\vec{r} = \frac{\rho_0}{3\epsilon_0}\int r\cdot dr = \frac{-\rho_0}{6\epsilon_0} r^2 + \text{const}$$

Outside

$r \ge R_B$

$$E\,4\pi r^2 = \frac{1}{\epsilon_0}\int_0^{R_B} \rho_0 4\pi r^2\,dr = \frac{4\pi\rho_0}{\epsilon_0}\frac{R_B^3}{3}$$

$$\vec{E} = \frac{\rho_0 R_B^3}{3\epsilon_0}\frac{1}{r^2}\hat{r} \qquad V_{out} = -\int \vec{E}\cdot d\vec{r} = \frac{\rho_0 R_B^3}{3\epsilon_0}\frac{1}{r} + \text{const.}$$



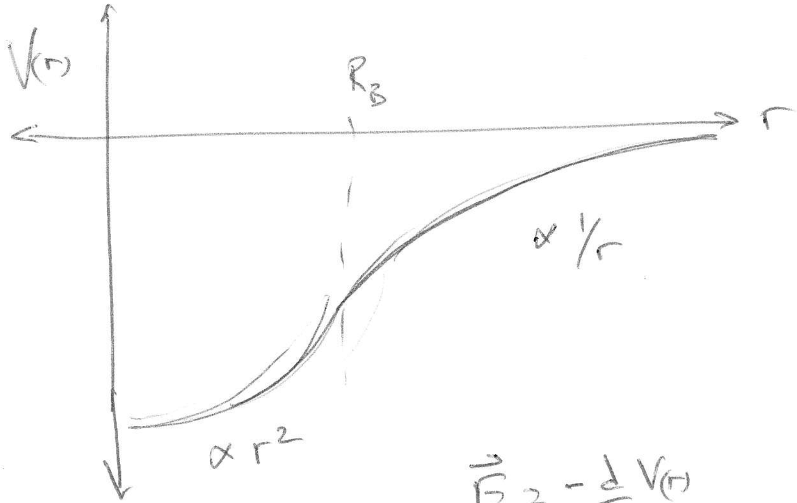
Boundary Conditions

$V_{in}(R_B) = V_{out}(R_B)$

$V_{out}(\infty) = 0$ so $V_{out} = \dfrac{\rho_0 R_B^3}{3\epsilon_0 r}$

$$\text{const} + \frac{-\rho_0}{6\epsilon_0}R_B^2 = \frac{2\rho_0 R_B^{\cancel{3}\,2}}{6\,\cancel{3}\epsilon_0 \cancel{R_B}} \cancel{+\text{const}}.$$

$$+\frac{1}{2}\frac{\rho_0 R_B^2}{\epsilon_0} = \text{const}$$



$$\vec{E} = -\frac{d}{dr}V(r)$$

$$\epsilon_0 = \frac{1}{4\pi k_0} = 8.85\times10^{-12}\,\frac{C^2}{Nm^2}$$

$$k_0 = 8.99\times10^{9}\,\frac{Nm^2}{C^2}$$

Inside

(b) $r_1 = 0.1\,m \quad r_2 = 0.2\,m$

$$\Delta V = \frac{-\rho_0}{6\epsilon_0} r^2 \Big|_{.1}^{.2} = \frac{-\rho_0}{6\epsilon_0}(.2^2 - .1^2)$$

$$= -4.3\times10^6\,V$$

Outside

$r_1 = .40\,m \quad r_2 = 0.60\,m$

$$\Delta V = \frac{\rho_0 R_B^3}{3\epsilon_0}\frac{1}{r}\Big|_{0.4}^{0.6}$$

$$= -6.4\times10^6\,V$$